



# Esercizi di consolidamento

Risolvi le seguenti equazioni.

1  $4x^4 - 5x^3 + 5x - 4 = 0$

$[S = \{\pm 1\}]$

2  $3x^3 - 4x^2 - 5x + 2$

$[S = \{-1, 2, \frac{1}{3}\}]$

3  $4x^4 - 13x^2 + 3 = 0$

$[S = \{\pm\sqrt{3}, \pm\frac{1}{2}\}]$

4  $\frac{4}{3}(x^2 + 2) = \frac{5x^2 - 1}{x^2}$

$[S = \{\pm 1, \pm\frac{\sqrt{3}}{2}\}]$

5  $\frac{2x-1}{x+2} - \frac{x-4}{x^2+1} = -\frac{1}{2}$

$[S = \{-1\}]$

6  $2x + \frac{x^2+2}{x^2-x} = x$

$[S = \{-\sqrt[3]{2}\}]$

7  $2\left(\frac{x-1}{x} - \frac{x^2+3}{x+1}\right) + 4x = 0$

$[S = \{1, -2 \pm \sqrt{3}\}]$

Risolvi le seguenti disequazioni.

8  $x^3 - 6x^2 + 12x - 8 \geq 0$

$[x \geq 2]$

9  $x^4 - 6x^2 + 8 > 0$

$[x < -2 \vee -\sqrt{2} < x < \sqrt{2} \vee x > 2]$

10  $(x-1)^3 + (x-2)^2 > 1$

$[-1 < x < 1 \vee x > 2]$

11  $x^3 - 3x^2 + 2x > 0$

$[0 < x < 1 \vee x > 2]$

12  $(4x^2 - 3)(x+2) > x^2 + 2x$

$[-2 < x < -\frac{3}{4} \vee x > 1]$

13  $2 - \frac{3}{4}(x-2)^3 < 2x + \frac{3}{4}$

$[x > 1]$

14  $\frac{x^2+x-2}{x^2(x-1)} > 0$

$[x > -2 \wedge x \neq 0 \wedge x \neq 1]$

15  $\frac{x^2(x-3)}{x+3} \leq 0$

$[-3 < x \leq 3]$

16  $\frac{x^3-4x}{x^2+1} > 0$

$[-2 < x < 0 \vee x > 2]$

17  $\frac{2+x}{x} - \frac{x^2+1}{x-1} + 3 < 0$

$[1 - \sqrt{2} < x < 0 \vee 1 < x < 2 \vee x > 1 + \sqrt{2}]$

18  $x + \frac{3x+1}{x-2} > \frac{x^2}{x^2-4}$

$[-2 < x < -1 \vee x > 2]$

19  $\frac{1}{x^2-x} + \frac{x}{x-1} > \frac{5}{2}$

$[-\frac{1}{3} < x < 0 \vee 1 < x < 2]$

Ripasso: equazioni e disequazioni

Risolvi i seguenti sistemi di disequazioni.

$$20 \quad \begin{cases} x^2 - 5x < 0 \\ \frac{1}{x} > 2 \end{cases} \quad \left[ 0 < x < \frac{1}{2} \right]$$

$$21 \quad \begin{cases} x > \frac{2}{x} \\ 3x^2 + 5x - 2 \leq 0 \end{cases} \quad [-\sqrt{2} < x < 0]$$

$$22 \quad \begin{cases} 9x^2 - 1 < 0 \\ \frac{5x+1}{3x} > 2x \end{cases} \quad \left[ -\frac{1}{3} < x < -\frac{1}{6} \vee 0 < x < \frac{1}{3} \right]$$

$$23 \quad \begin{cases} \frac{x+1}{2x-1} > \frac{3}{1-4x^2} \\ 2-x < 0 \end{cases} \quad [x > 2]$$

$$24 \quad \begin{cases} 3-x^2 > 0 \\ 3-x > x^2 - \frac{3}{2}x \\ \frac{4}{x^2} > 1 \end{cases} \quad \left[ -\frac{3}{2} < x < 0 \vee 0 < x < \sqrt{3} \right]$$

$$25 \quad \begin{cases} 4x^2 + x > 0 \\ \frac{3x-1}{x} > 0 \\ \frac{1}{x} \geq \frac{4}{3}x \end{cases} \quad \left[ x \leq -\frac{\sqrt{3}}{2} \vee \frac{1}{3} < x \leq \frac{\sqrt{3}}{2} \right]$$

Risolvi le seguenti equazioni irrazionali.

$$26 \quad 1 - \sqrt{6x+3} = 2 - x \quad [S = \{3\sqrt{2} + 4\}]$$

$$27 \quad \sqrt{x^2 - 3x} + 2x = 0 \quad [S = \{-1, 0\}]$$

$$28 \quad \sqrt{x^2 + 13x - 9} - 5x = 2 - 4x \quad \left[ S = \left\{ \frac{13}{9} \right\} \right]$$

$$29 \quad 2\sqrt{3x-4} = \sqrt{x^2-1} \quad [S = \{6 \pm \sqrt{21}\}]$$

$$30 \quad \sqrt{\frac{x-1}{x}} + 3 = 0 \quad [S = \emptyset]$$

$$31 \quad 4\sqrt{1-x} = 1 + \sqrt{9-x} \quad [S = \{0\}]$$

$$32 \quad \sqrt{2x+1} = \sqrt{x} + 1 \quad [S = \{0, 4\}]$$

$$33 \quad \sqrt[3]{x^2 - 2x - 3} = \sqrt[3]{x+1} \quad [S = \{-1, 4\}]$$

$$34 \quad \sqrt[3]{6-x^2} = \sqrt[3]{x^2+5} \quad \left[ S = \left\{ \pm \frac{\sqrt{2}}{2} \right\} \right]$$

Risolvi le seguenti disequazioni irrazionali.

$$35 \quad \sqrt{2x} > x \quad [0 < x < 2]$$

$$36 \quad \sqrt{x^2 - 3x} > 2 \quad [x < -1 \vee x > 4]$$

$$37 \quad \sqrt{x^2 - 8x + 15} > 2x - 1 \quad \left[ x < \frac{-2 + \sqrt{46}}{3} \right]$$

$$38 \quad \sqrt{\frac{x^2 + 1}{x - 3}} \geq 2 \quad [x > 3]$$

$$39 \quad \sqrt{x + 3} < 2x - 4 \quad \left[ x > \frac{13}{4} \right]$$

$$40 \quad 2x - 1 > \sqrt{x} \quad [x > 1]$$

$$41 \quad \sqrt{x^2 + 3} > x + 3 \quad [x < -1]$$

$$42 \quad \sqrt{3x^2 - 4x + 1} < x + 2 \quad \left[ \frac{4 - \sqrt{22}}{2} < x \leq \frac{1}{3} \vee 1 \leq x < \frac{4 + \sqrt{22}}{2} \right]$$

$$43 \quad 2 + \sqrt{5 - x^2} > x + 3 \quad [-\sqrt{5} \leq x < 1]$$

$$44 \quad 2x - 5 > \sqrt{x^2 - 6x + 10} \quad [x > 3]$$

Risolvi le seguenti equazioni con i moduli.

$$45 \quad |2x - x^2| + 4x = 3 \quad [S = \{-3, 3 - \sqrt{6}\}]$$

$$46 \quad 4 + |x| = x^2 + 2 \quad [S = \{\pm 2\}]$$

$$47 \quad |x^2 - 3x| + x = 4 \quad [S = \{2, 1 \pm \sqrt{5}\}]$$

$$48 \quad |4x^2 + x - 5| + 3 = 0 \quad [S = \emptyset]$$

$$49 \quad \left| \frac{x^2 + 4}{x} \right| = 5 \quad [S = \{\pm 1, \pm 4\}]$$

$$50 \quad 2x - \left| 1 - \frac{1}{x} \right| + 4 = 0 \quad \left[ S = \left\{ -1, -\frac{1}{2}, \frac{-5 + \sqrt{33}}{4} \right\} \right]$$

$$51 \quad \frac{x+1}{x} - \frac{5}{2} = |x| \quad \left[ S = \left\{ \frac{1}{2} \right\} \right]$$

$$52 \quad \frac{3}{4} - |x^2 + 5x| = x + 3 \quad \left[ S = \left\{ -\frac{9}{2}, -\frac{6 + 3\sqrt{3}}{2} \right\} \right]$$

Risolvi le seguenti disequazioni con i moduli.

$$53 \quad \left| \frac{1}{2}x^2 - x \right| > 1 \quad \left| x^2 - \frac{3}{4} \right| < \frac{3}{2} \quad \left[ x < 1 - \sqrt{3} \vee x > 1 + \sqrt{3}; -\frac{3}{2} < x < \frac{3}{2} \right]$$

$$54 \quad |5x - 6x^2| + 3 > 0 \quad |3x^2 + 2x - 1| - 4 < 0 \quad \left[ S = \mathbb{R}; -\frac{5}{3} < x < 1 \right]$$

$$55 \quad 2 - |x + 4x^2| > x - 4 \quad x^2 - 2 \left| x^2 + \frac{1}{2}x \right| > -2 \quad \left[ -\frac{3}{2} < x < 1; -2 < x < 1 \right]$$

$$56 \quad 2x - 5 \geq \left| \frac{1}{2}x^2 - 1 \right| + x^2 - 6 \quad |4x + 1 - x^2| - 3x \geq x^2 - 2 \quad \left[ 0 \leq x \leq 2; x \leq \frac{3}{2} \right]$$

$$57 \quad \frac{|x^2 - 3| + 2}{x} > 0 \quad \frac{x + 2}{|x^2 - 4| + 1} > \frac{1}{4} \quad \left[ x > 0; -1 < x < 2 + \sqrt{15} \right]$$

Ripasso: equazioni e disequazioni