Esercizi di consolidamento

Stabilisci per quali valori delle lettere le seguenti frazioni algebriche hanno significato.

1 ┌ esercizio **guidato**

a. $\dfrac{3x - y}{2x^2 y}$

La frazione ha significato se il denominatore è diverso da zero e ciò accade se $x \neq 0$ e $y \neq 0$.

b. $\dfrac{x - 3a}{x^2 - x}$

Scomponiamo il denominatore: $\dfrac{x - 3a}{x(x - 1)}$

La frazione ha significato quando nessuno dei fattori al denominatore è zero, quindi se $x \neq 0$ e $x \neq 1$.

2 **a.** $\dfrac{3x + 1}{x}$; **b.** $\dfrac{a + 4}{3a}$ $[\textbf{a.}\ x \neq 0;\ \textbf{b.}\ a \neq 0]$

3 **a.** $\dfrac{2a + b}{a - 1}$; **b.** $\dfrac{x^2 - y}{x^2 + x}$ $[\textbf{a.}\ a \neq 1;\ \textbf{b.}\ x \neq 0 \ \wedge\ x \neq -1]$

4 **a.** $\dfrac{x^2 + 1}{2x - 4}$; **b.** $\dfrac{3a - b}{a^2 - 4b^2}$ $[\textbf{a.}\ x \neq 2;\ \textbf{b.}\ a \neq \pm 2b]$

Semplifica le seguenti frazioni.

5 ┌ esercizio **guidato**

$\dfrac{4x^5 y^2}{3x^3 y^5}$

Per semplificare questa frazione, che è il rapporto fra due monomi interi, basta applicare le proprietà delle potenze:

$$\dfrac{4x^5 y^2}{3x^3 y^5} = \dfrac{4x^2}{3y^3}$$

6 $\dfrac{3a^3 x^2 z}{9az^2}$; $\dfrac{8x^6}{4x^5}$; $\dfrac{2b^6 y^4}{12b^6 y}$ $\left[\dfrac{a^2 x^2}{3z};\ 2x;\ \dfrac{y^3}{6}\right]$

7 $\dfrac{12a^7 b^9 c^{15}}{24b^8 c^{11}}$; $\dfrac{8b^4 ac^3}{-24b^5 c^5}$; $\dfrac{2x^{18} y^{16}}{4x^{10} y^4}$ $\left[\dfrac{a^7 bc^4}{2};\ -\dfrac{a}{3bc^2};\ \dfrac{x^8 y^{12}}{2}\right]$

8 ┌ esercizio **guidato**

$\dfrac{3x + 15}{x^2 - 25}$

Scomponiamo in fattori i polinomi al numeratore e al denominatore $\dfrac{3x + 15}{x^2 - 25} = \dfrac{3(x + 5)}{(x + 5)(x - 5)}$

Semplifichiamo la frazione applicando la proprietà invariantiva: $\dfrac{3\cancel{(x + 5)}}{\cancel{(x + 5)}(x - 5)} = \dfrac{3}{x - 5}$

9 $\dfrac{3a^2y - 6ay^2}{4a^2y - 2a^3};$ $\dfrac{x^2y^3z^2}{x^2 + y^3 + z^2}$ $\left[-\dfrac{3y}{2a}\,;\ \text{irriducibile}\right]$

10 $\dfrac{2 - 2y - y^2 + y^3}{1 - y + 3y^2 - 3y^3};$ $\dfrac{a^3 - 8 - 6a^2 + 12a}{a^3 - 8}$ $\left[\dfrac{2 - y^2}{1 + 3y^2}\,;\ \dfrac{(a-2)^2}{a^2 + 2a + 4}\right]$

11 $\dfrac{a^3 - a^2 + 3a^2(a-1)}{16a^2 + 16 - 32a};$ $\dfrac{7z^2 - 28}{2z^2 + 8 + 8z}$ $\left[\dfrac{a^2}{4(a-1)}\,;\ \dfrac{7(z-2)}{2(z+2)}\right]$

12 $\dfrac{100 - 81t^2}{-300t - 243t^3 + 540t^2};$ $\dfrac{25 - 9x^2}{25 + 9x^2 + 30x}$ $\left[\dfrac{10 + 9t}{3t(9t - 10)}\,;\ \dfrac{5 - 3x}{5 + 3x}\right]$

13 $\dfrac{(3x-1)^2 - (x-3)^2}{x^3 - x^2 + 4x - 4};$ $\dfrac{24x^2 - 6}{8x^2 - 8x + 2}$ $\left[\dfrac{8(x+1)}{x^2 + 4}\,;\ \dfrac{3(2x+1)}{2x - 1}\right]$

14 $\dfrac{y^3 - 3ay^2 + 3a^2y - a^3}{y^3 - a^3};$ $\dfrac{a^2 + 2a + 1}{4a^2 + 28a + 24}$ $\left[\dfrac{(y-a)^2}{y^2 + ay + a^2}\,;\ \dfrac{a+1}{4(a+6)}\right]$

15 $\dfrac{ax^2 - 4a}{6x^2 + 24x + 24};$ $\dfrac{x^6 + x^3y^4}{x^6 - y^8}$ $\left[\dfrac{a(x-2)}{6(x+2)}\,;\ \dfrac{x^3}{x^3 - y^4}\right]$

16 $\dfrac{x^2 + 9 + 6x}{-2x^2 + 6x + 36};$ $\dfrac{x^2 + 1 + 4y^2 - 2x + 4xy - 4y}{x^2 + 4y^2 + 4xy - 1}$ $\left[\dfrac{x+3}{2(6-x)}\,;\ \dfrac{x - 1 + 2y}{x + 2y + 1}\right]$

17 $\dfrac{3a^3 - a + 1 - 3a^2}{9a^4 - 1};$ $\dfrac{x^2 - 2xy - 15y^2}{x^2 - 8xy + 15y^2}$ $\left[\dfrac{a-1}{3a^2 + 1}\,;\ \dfrac{x + 3y}{x - 3y}\right]$

18 $\dfrac{2a - 4}{4a^3 - 32 - 24a^2 + 48a};$ $\dfrac{z^3 + 3z^2 - 4z - 12}{3z^2 + 15z + 18}$ $\left[\dfrac{1}{2(a-2)^2}\,;\ \dfrac{z-2}{3}\right]$

19 $\dfrac{(2a + 6b)^3 - 8a^3}{72ab};$ $\dfrac{27 - x^3}{x^2 - 9}$ $\left[\dfrac{a^2 + 3ab + 3b^2}{a}\,;\ -\dfrac{x^2 + 3x + 9}{x + 3}\right]$

20 $\dfrac{x^2 + x - 2}{x^2 + 3x + 2}$ $\dfrac{ax^2 - a}{a^2x - a^2}$ $\left[\dfrac{x-1}{x+1}\,;\ \dfrac{x+1}{a}\right]$

21 $\dfrac{3x^3 - 3y^{12}}{9(x^2 + xy^4 + y^8)}$ $\dfrac{a^2x^6 + a^2x^3 - 6a^2}{ax^6 - ax^3 - 2a}$ $\left[\dfrac{x - y^4}{3}\,;\ \dfrac{a(x^3 + 3)}{x^3 + 1}\right]$

Riduci allo stesso denominatore i seguenti gruppi di frazioni.

22 **esercizio guidato**

$\dfrac{2x}{3x^2 - 3y^2}$ $\dfrac{x+y}{x-y}$ $\dfrac{3y}{x^2 - xy}$

Scomponiamo i denominatori delle tre frazioni:

$\dfrac{2x}{3(x-y)(x+y)}$ $\dfrac{x+y}{x-y}$ $\dfrac{3y}{x(x-y)}$

Nessuna delle frazioni può essere semplificata; calcoliamo dunque il *m.c.m.* fra i denominatori:
$3x(x-y)(x+y)$

Le frazioni date sono dunque equivalenti a:

$\dfrac{2x^2}{3x(x-y)(x+y)}$ $\dfrac{3x(x+y)^2}{3x(x-y)(x+y)}$ $\dfrac{9y(x+y)}{3x(x-y)(x+y)}$

23 $\dfrac{2}{x^2}$ $\qquad -\dfrac{5}{3y^3}$ $\qquad \dfrac{3}{6xy^2}$ $\qquad \left[\dfrac{12y^3}{6x^2y^3}; -\dfrac{10x^2}{6x^2y^3}; \dfrac{3xy}{6x^2y^3}\right]$

24 $\dfrac{2a-2b}{6(a+b)}$ $\qquad \dfrac{3a}{9a^2+9b^2+18ab}$ $\qquad \dfrac{5b}{10a+10b}$ $\qquad \left[\dfrac{2(a^2-b^2)}{6(a+b)^2}; \dfrac{2a}{6(a+b)^2}; \dfrac{3b(a+b)}{6(a+b)^2}\right]$

(Suggerimento: semplifica prima le frazioni quando è possibile)

25 $\dfrac{3-2x}{-2x^2+5x-3}$ $\qquad \dfrac{5x-10}{x^2-x-2}$ $\qquad \dfrac{x^2+x-6}{x^2+4x+3}$ $\qquad \left[\dfrac{x+1}{x^2-1}; \dfrac{5(x-1)}{x^2-1}; \dfrac{(x-2)(x-1)}{x^2-1}\right]$

26 $\dfrac{a^2-b^2}{a^2-ab+3a-3b}$ $\qquad \dfrac{3a}{a-2}$ $\qquad \dfrac{2a+4}{3a^2-12}$ $\quad \left[\dfrac{3(a-2)(a+b)}{3(a-2)(a+3)}; \dfrac{9a(a+3)}{3(a-2)(a+3)}; \dfrac{2(a+3)}{3(a-2)(a+3)}\right]$

27 $\dfrac{3+2a}{4a^2+9+12a}$ $\qquad \dfrac{5a}{10a+15}$ $\qquad \dfrac{-7a^2}{4a^2-9}$ $\quad \left[\dfrac{2a-3}{(2a-3)(2a+3)}; \dfrac{a(2a-3)}{(2a-3)(2a+3)}; \dfrac{-7a^2}{(2a-3)(2a+3)}\right]$

28 $\dfrac{x-2}{-6x+6y}$ $\qquad \dfrac{y+1}{x^2-y^2}$ $\qquad \dfrac{2}{3x+3y}$ $\qquad \left[\dfrac{(2-x)(x+y)}{6(x^2-y^2)}; \dfrac{6(y+1)}{6(x^2-y^2)}; \dfrac{4(x-y)}{6(x^2-y^2)}\right]$

29 $\dfrac{2-3x}{9x^2+4-12x}$ $\qquad \dfrac{4y}{24x-16}$ $\qquad \dfrac{-3x^2}{9x^2-4}$ $\qquad \left[\dfrac{2(2+3x)}{2(4-9x^2)}; \dfrac{-y(2+3x)}{2(4-9x^2)}; \dfrac{6x^2}{2(4-9x^2)}\right]$

30 $\dfrac{a^2+b^2}{a^3-b^3}$ $\qquad \dfrac{3a}{a^2+ab+b^2}$ $\qquad \dfrac{2a+2b}{a^2-b^2}$ $\qquad \left[\dfrac{a^2+b^2}{a^3-b^3}; \dfrac{3a(a-b)}{a^3-b^3}; \dfrac{2(a^2+ab+b^2)}{a^3-b^3}\right]$

31 $\dfrac{2-3a}{9a^2-12a+4}$ $\qquad \dfrac{4b}{24a-16}$ $\qquad \dfrac{9a^2}{9a^2-4}$ $\qquad \left[\dfrac{-2(3a+2)}{2(9a^2-4)}; \dfrac{b(3a+2)}{2(9a^2-4)}; \dfrac{18a^2}{2(9a^2-4)}\right]$

32 $\dfrac{a^2+1}{x^3+6x^2+12x+8}$ $\quad \dfrac{a^2-3a}{ax-3x+2a-6}$ $\quad \dfrac{2a^3}{2x^2+8x+8}$ $\qquad \left[\dfrac{a^2+1}{(x+2)^3}; \dfrac{a(x+2)^2}{(x+2)^3}; \dfrac{a^3(x+2)}{(x+2)^3}\right]$

33 $\dfrac{5x-10y}{3x^2-3xy-6y^2}$ $\quad \dfrac{5x^2-5xy+5y^2}{x^3+y^3}$ $\quad x-y$ $\qquad \left[\dfrac{5}{3(x+y)}; \dfrac{15}{3(x+y)}; \dfrac{3x^2-3y^2}{3(x+y)}\right]$

Esegui le addizioni e le sottrazioni fra le seguenti frazioni algebriche.

34 **esercizio guidato**

$\dfrac{10}{x-2}+\dfrac{x+2}{x}+\dfrac{2}{3x^2-x}$

Scomponiamo il denominatore della terza frazione: $\quad \dfrac{10}{x-2}+\dfrac{x+2}{x}+\dfrac{2}{x(3x-1)}$

Il *m.c.m.* fra i denominatori è $\quad x(x-2)(3x-1)$

La somma è quindi: $\quad \dfrac{10x(3x-1)+(x+2)(x-2)(3x-1)+2(x-2)}{x(x-2)(3x-1)}$

Sviluppando i calcoli al numeratore otteniamo: $\quad \dfrac{3x^3+29x^2-20x}{x(x-2)(3x-1)}$

Scomponiamo adesso il numeratore e semplifichiamo la frazione:

$$\dfrac{x(3x^2+29x-20)}{x(x-2)(3x-1)}=\dfrac{3x^2+29x-20}{3x^2-7x+2}$$

35 $\dfrac{y}{2y-3} - 3;$ $\qquad$ $1 - \dfrac{x+a}{x-a}$ $\qquad$ $\left[\dfrac{9-5y}{2y-3};\ \dfrac{2a}{a-x}\right]$

36 $\dfrac{1}{z-3} + \dfrac{4}{5z-15};$ $\qquad$ $\dfrac{3}{ab} - \dfrac{2}{ab-1}$ $\qquad$ $\left[\dfrac{9}{5(z-3)};\ \dfrac{ab-3}{ab(ab-1)}\right]$

37 $\dfrac{3}{7a+7} - \dfrac{a-5}{14a+14};$ $\qquad$ $\dfrac{a}{x+1} + \dfrac{a+b}{x^2-1} - \dfrac{b}{x-1}$ $\qquad$ $\left[\dfrac{11-a}{14(a+1)};\ \dfrac{x(a-b)}{(x^2-1)}\right]$

38 $2 - \dfrac{a+3b}{a-3b} - \dfrac{a-3b}{a+3b}$ $\qquad$ $\left[\dfrac{-36b^2}{a^2-9b^2}\right]$

39 $\dfrac{3a-x}{3x+9a} + \dfrac{x-a}{5x+15a} + \dfrac{2}{15}$ $\qquad$ $\left[\dfrac{6a}{5(x+3a)}\right]$

40 $\dfrac{4a^2}{a^3-a} - \dfrac{1}{3a+3} + \dfrac{4}{1-a}$ $\qquad$ $\left[\dfrac{a+11}{3(1-a^2)}\right]$

41 $-\dfrac{7}{x+3y} - \left(\dfrac{42y}{x^2-9y^2} - \dfrac{7}{x-3y}\right)$ $\qquad$ $[0]$

42 $\dfrac{y+z}{(x-y)(x-z)} + \dfrac{x+z}{(x-y)(y-z)} - \dfrac{x+y}{(x-z)(y-z)}$ $\qquad$ $\left[\dfrac{2(y+z)}{(x-y)(x-z)}\right]$

43 $\dfrac{2x(x+1)}{x^3-8} + \dfrac{x^2}{2x^2+4x+8} - \dfrac{x}{4-2x}$ $\qquad$ $\left[\dfrac{x}{x-2}\right]$

44 $\dfrac{x^2-1}{x^2-2x+1} + \dfrac{1-x^2}{x^2+2x+1} - \dfrac{4x^2}{x^2-1}$ $\qquad$ $\left[-\dfrac{4x}{x+1}\right]$

45 $\dfrac{x+2}{4} - \dfrac{x^3-8-6x^2+12x}{4x^2+16+16x} - \dfrac{3x^2-1}{x^2+4+4x}$ $\qquad$ $\left[\dfrac{5}{(x+2)^2}\right]$

46 $\dfrac{a^3-y^3}{a^3+3ay^2-3a^2y-y^3} + \dfrac{ay}{a^2-2ay+y^2} + \dfrac{a+y}{y-a}$ $\qquad$ $\left[\dfrac{2y(a+y)}{(a-y)^2}\right]$

47 $\left(\dfrac{3x-2}{x^2-4x+3} - \dfrac{1-x}{x^2+x-2}\right) - \left[\dfrac{5x-4}{-x^2+4x-3} + \dfrac{8x^2+7x-3}{(x^2-4x+3)(x+2)}\right]$ $\qquad$ $\left[\dfrac{1}{x-1}\right]$

48 $\dfrac{1}{x} + \dfrac{2(x-1)}{1+x} - \left[\dfrac{x}{x^2-1} - \dfrac{2x+1}{x^3-x} - \dfrac{(x+1)^2}{x^2-2x+1} + \dfrac{3\left(x^3-\dfrac{5}{3}\right)-5x(x-3)}{x^3-x^2-x+1}\right]$ $\qquad$ $\left[\dfrac{2}{(x+1)}\right]$

49 $\dfrac{a^2-1}{a+b} + \dfrac{a-1}{a^2-2ab-3b^2} - \left[\dfrac{a+1}{a^2-9b^2} + \dfrac{2ab-2a-4b}{(a^2-9b^2)(a+b)}\right]$ $\qquad$ $\left[\dfrac{a^2-1}{a+b}\right]$

50 $\dfrac{a^2}{a-b} + \dfrac{b+2}{a+b+1} - \left[\dfrac{2a-3b-2b^2}{a(a+1)-b(b+1)} + \dfrac{b}{a-b}\right]$ $\qquad$ $\left[\dfrac{a^2}{a-b}\right]$

51 $\dfrac{x}{x+1} + \dfrac{1}{3x-2} - \left(1 + \dfrac{3}{3x^2+x-2}\right)$ $\qquad$ $\left[\dfrac{-2x}{(x+1)(3x-2)}\right]$

52 $\dfrac{a(x+2y)}{x^2+2yx+y^2} - \left(\dfrac{ax^2+3ay^2+4axy}{(x+y)^3} - \dfrac{a^2}{ax+ay}\right) + \dfrac{ay}{(x+y)^2}$ $\qquad$ $\left[\dfrac{a}{x+y}\right]$

53 $\dfrac{x^2-4a^2}{x^2+4ax+4a^2} - \left[\dfrac{ax+a^2-1}{a(x+a)} - \left(\dfrac{4a^2-1}{ax+2a^2} - \dfrac{1}{x^2+3ax+2a^2}\right)\right]$ $\qquad$ $[0]$

54 $\dfrac{x}{x+2} - \left[\dfrac{a+2}{x^2+2x-ax-2a} - \left(\dfrac{1}{x-a} - 2\right)\right]$ $\qquad\qquad \left[-\dfrac{x+3}{x+2}\right]$

55 $\dfrac{ax^2}{x^2+2yx+y^2} + \dfrac{1+axy^2}{(x+y)^3} - \left(\dfrac{ax}{x+y} - \dfrac{axy}{x^2+2xy+y^2}\right)$ $\qquad \left[\dfrac{1+axy^2}{(x+y)^3}\right]$

56 $\dfrac{2x-1}{ax-1} + \left[\dfrac{x}{ax-2} - \left(\dfrac{2-2x-ax}{a^2x^2-3ax+2} + \dfrac{2x^2}{ax^2-2x}\right)\right]$ $\qquad \left[\dfrac{x}{ax-2}\right]$

57 $\dfrac{x}{x^2+2xy+y^2} - \left[\dfrac{x^2-y^2}{(x+y)^3} - \left(\dfrac{y}{xy+y^2} - \dfrac{2y}{x^2+2xy+y^2}\right)\right]$ $\qquad \left[\dfrac{x}{(x+y)^2}\right]$

Esegui le seguenti moltiplicazioni fra frazioni algebriche e calcola il valore delle espressioni.

58 **esercizio guidato**

> $\dfrac{x^2+x-2}{3x+9} \cdot \dfrac{2x+6}{4x-4}$
>
> Scomponiamo i polinomi delle due frazioni: $\dfrac{(x+2)(x-1)}{3(x+3)} \cdot \dfrac{2(x+3)}{4(x-1)}$
>
> Semplifichiamo: $\dfrac{(x+2)\cancel{(x-1)}}{3\cancel{(x+3)}} \cdot \dfrac{\cancel{2}\cancel{(x+3)}}{\underset{2}{\cancel{4}}\cancel{(x-1)}}$
>
> Moltiplichiamo i numeratori e i denominatori rimasti: $\dfrac{x+2}{6}$

59 $\dfrac{a^2b^3}{x^3z^3} \cdot \dfrac{x^2z^2}{a^2b^2};$ $\qquad\qquad \dfrac{3bc^2}{7a} \cdot \dfrac{14a^2}{6b^2c}$ $\qquad\qquad \left[\dfrac{b}{xz}; \ \dfrac{ac}{b}\right]$

60 $\dfrac{4a^2}{b^3c^2}\left(-\dfrac{b^2c}{a^2}\right)\left(-\dfrac{1}{2}ac\right);$ $\qquad \left(-\dfrac{y^3}{16}\right) \cdot \dfrac{8a^4x^2}{y} \cdot \dfrac{4x}{a^3y^2}$ $\qquad \left[\dfrac{2a}{b}; -2ax^3\right]$

61 $\dfrac{1}{12a^2}\left(-\dfrac{7a^2b^2}{5}\right)\left(-\dfrac{60a^3}{b^3}\right);$ $\qquad \dfrac{2x-4}{x} \cdot \dfrac{3x^3}{x^2-4} \cdot \dfrac{x+2}{9}$ $\qquad \left[\dfrac{7a^3}{b}; \ \dfrac{2}{3}x^2\right]$

62 $\dfrac{ax+a-x-1}{ax-a-2x+2} \cdot \dfrac{ax-2a-2x+4}{x^2-x-2}$ $\qquad\qquad \left[\dfrac{a-1}{x-1}\right]$

63 $\dfrac{9}{3x-12} \cdot \dfrac{x^2-16}{3x^2+48+24x}$ $\qquad\qquad \left[\dfrac{1}{x+4}\right]$

64 $\dfrac{2y^2-18}{b-4} \cdot \dfrac{b^2-8b+16}{2y-6}$ $\qquad\qquad [(b-4)(y+3)]$

65 $\dfrac{x^2-y^2}{x^2+y^2} \cdot \dfrac{x^2}{y-x} \cdot \dfrac{1}{x^2+xy}$ $\qquad\qquad \left[-\dfrac{x}{x^2+y^2}\right]$

66 $\dfrac{x^4-81}{x^3+27+9x^2+27x} \cdot \dfrac{x^2+x-6}{27+3x^2} \cdot \dfrac{6x+18}{2x^2-8}$ $\qquad \left[\dfrac{x-3}{x+2}\right]$

67 $\dfrac{a^2-3ab+2b^2}{a^3+4a^2b+4ab^2} \cdot \dfrac{ab+2b^2}{9a^2b-36ab^2+36b^3} \cdot \dfrac{3a^3-12ab^2}{2b-2a}$ $\qquad \left[-\dfrac{1}{6}\right]$

68 $\dfrac{4a^2 - 16}{a^3 - 8} \cdot \dfrac{a^2x + 4x + 2ax - 4 - 2a - a^2}{3x - 2 - x^2} \cdot \dfrac{x^2 - 5x + 6}{2a + 4}$ $\qquad\qquad [2(3 - x)]$

69 $\dfrac{x^2 - 4}{x} \cdot \dfrac{x^2 - x}{2x^2 - 2} \cdot \left(-\dfrac{x^2 - x - 2}{x^2 + 4x + 4}\right)$ $\qquad\qquad \left[-\dfrac{(x - 2)^2}{2(x + 2)}\right]$

70 $\dfrac{x + x^3}{3ay^2} \cdot \dfrac{2x - 4}{x^4} \cdot \left(-\dfrac{\frac{3}{2}x + 3}{ax^2 + a}\right) \cdot \dfrac{a^2x}{2 - x}$ $\qquad\qquad \left[\dfrac{x + 2}{x^2y^2}\right]$

71 $\dfrac{a^2b^2 + 3ab + 2}{ab^3} \cdot \left(-\dfrac{a^2(ab + 2)}{a^2b^2 - 2ab - 3}\right) \cdot \left(-\dfrac{b^3}{a^3b^2 + 4a^2b + 4a}\right)$ $\qquad\qquad \left[\dfrac{1}{ab - 3}\right]$

72 $\dfrac{a^5 - 8a^3b^2 + 16ab^4}{9bx^4} \cdot \left(-\dfrac{15x^3y}{a^2bc^2 - 4b^3c^2}\right) \cdot \dfrac{3b^2c^2}{5ay} \cdot \dfrac{x}{2a + 4b}$ $\qquad\qquad \left[\dfrac{2b - a}{2}\right]$

73 $\left(\dfrac{1}{x - 2} + 1\right)\left(1 - \dfrac{1}{x - 1}\right)$ $\qquad\qquad [1]$

(Suggerimento: esegui prima le operazioni dentro le parentesi tonde e poi moltiplica)

74 $\dfrac{3 - a}{a^2 - 2a - 3}\left(\dfrac{a^2 - 2a}{a - 3} + \dfrac{3}{3 - a}\right)$ $\qquad\qquad [-1]$

75 $\dfrac{1 - 4x^2}{4x + 18}\left(\dfrac{4}{2x + 1} - \dfrac{5}{2x - 1}\right)$ $\qquad\qquad \left[\dfrac{1}{2}\right]$

76 $\left(\dfrac{3a}{a^2 - 4a + 3} + \dfrac{4}{1 - a}\right)\left(\dfrac{a - 6}{12 - a} + \dfrac{1}{3}\right)$ $\qquad\qquad \left[\dfrac{2}{3(a - 1)}\right]$

77 $\left(\dfrac{1}{a - 1} - \dfrac{1}{a^2 - 1}\right) \cdot \left(\dfrac{1 + a}{1 - a} - \dfrac{1 - a}{a + 1}\right) \cdot \left(a - 2 + \dfrac{1}{a}\right)$ $\qquad\qquad \left[-\dfrac{4a}{(a + 1)^2}\right]$

78 $\dfrac{1}{y - 4}\left(\dfrac{1}{y - 4} + \dfrac{y}{4 - y^2}\right)\left(-\dfrac{2 - y}{4} + \dfrac{5 - 2y}{y + 2}\right)\left(y + 4 + \dfrac{4}{y}\right)$ $\qquad\qquad \left[\dfrac{y - 1}{y^2 - 2y}\right]$

Semplifica le seguenti espressioni nelle quali compaiono potenze di frazioni algebriche.

79 $\left(\dfrac{a^2b - ab^2}{a + b}\right)^2 \cdot \left(\dfrac{ab - b^2}{a + b}\right)^{-3};$ $\qquad\left(\dfrac{x - 3}{x + 3}\right)^2\left(\dfrac{x^2 - 9}{x + 3}\right)^{-2}$ $\qquad \left[\dfrac{a^2(a + b)}{b(a - b)};\ \dfrac{1}{(x + 3)^2}\right]$

80 $\left(\dfrac{5x + 10}{x - 2}\right)^3\left(\dfrac{15x + 30}{3x - 6}\right)^4;$ $\qquad\left(\dfrac{x - 3y}{x + y}\right)^3\left(\dfrac{x^2 - 9y^2}{x^2 + 4xy + 3y^2}\right)^2$ $\qquad \left[\dfrac{5^7(x + 2)^7}{(x - 2)^7};\ \dfrac{(x - 3y)^5}{(x + y)^5}\right]$

81 $\dfrac{4y^4}{4y^2 - 1} \cdot \left(\dfrac{3y - 1}{y}\right)^2\left(4 - \dfrac{1}{y^2}\right)$ $\qquad\qquad [4(3y - 1)^2]$

82 $\left(\dfrac{x^2 - 1}{x + 1}\right)^{-2} \cdot \dfrac{x^2 - 1}{x^2 + 1} \cdot \left[\dfrac{(x - 1)^2}{x^4 - 1}\right]^{-1}$ $\qquad\qquad \left[\left(\dfrac{x + 1}{x - 1}\right)^2\right]$

83 $\left(\dfrac{a + b}{a - 2b} - 1\right)^2\left(a - \dfrac{4b^2}{a}\right)^2\left(\dfrac{a - 2b}{a + 2b} + 1\right)^2$ $\qquad\qquad [36b^2]$

84 $\left(\dfrac{y^2 - 2y + 4}{y^3} - \dfrac{1}{y + 2}\right)^3\left(\dfrac{14 - 9y}{4 - y^2} + \dfrac{1}{2 - y}\right)^2$ $\qquad\qquad \left[\dfrac{1}{y^9}\left(\dfrac{8}{2 + y}\right)^5\right]$

Esegui le seguenti divisioni fra frazioni algebriche e calcola il valore delle espressioni.

85 | esercizio **guidato**

$$\frac{6x^2 + 3x - 9}{x^2 + ax - 2a^2} : \frac{6ax - 6x - 6a + 6x^2}{x^2 + 3ax + 2a^2}$$

Scomponiamo i polinomi che compongono le due frazioni e contemporaneamente trasformiamo la divisione in una moltiplicazione:

$$\frac{\cancel{3}(x-1)(2x+3)}{(x-a)\cancel{(x+2a)}} \cdot \frac{\cancel{(x+2a)}\,\cancel{(x+a)}}{\underset{2}{\cancel{6}}(x-1)\,\cancel{(x+a)}} = \frac{2x+3}{2(x-a)}$$

86 $\dfrac{x^3 y^2}{2x} : \dfrac{xy^3}{4x^2};$ $\qquad -\dfrac{1}{3} x^5 y^3 : \dfrac{x^4 y^4}{9}$ $\qquad \left[\dfrac{2x^3}{y}; \ -\dfrac{3x}{y}\right]$

87 $-\dfrac{32x^5 y^3}{25z^2} : \left(-\dfrac{64x^3 y^3}{15z^4}\right);$ $\qquad 1 : \left(-\dfrac{3ab}{x^3}\right)^2$ $\qquad \left[\dfrac{3}{10}z^2 x^2; \ \dfrac{x^6}{9a^2 b^2}\right]$

88 $\dfrac{4y}{x+y} : \dfrac{3y^2}{2x+2y};$ $\qquad \dfrac{a-2b}{9b^2} : \dfrac{6b-3a}{27b}$ $\qquad \left[\dfrac{8}{3y}; \ -\dfrac{1}{b}\right]$

89 $\dfrac{7ab}{a+b} : \dfrac{21a}{3a+3b};$ $\qquad -\dfrac{5x^2}{x+2y} : \dfrac{3x}{3x+6y}$ $\qquad [b; \ -5x]$

90 $\dfrac{x^2+5}{3xy} : \dfrac{2x^2+10}{x};$ $\qquad \dfrac{a+b}{3} : \dfrac{a^2+2ab+b^2}{9a}$ $\qquad \left[\dfrac{1}{6y}; \ \dfrac{3a}{a+b}\right]$

91 $\dfrac{8x^3-1}{7x-3-2x^2} : \dfrac{(2x+1)^2-2x}{9-x^2}$ $\qquad [x+3]$

92 $\dfrac{(3y-a)2x^2-9yz^2+3az^2}{4x^2-6z^2} : \dfrac{9y^2+a^2-6ay}{8}$ $\qquad \left[\dfrac{4}{3y-a}\right]$

93 $\dfrac{x^2+2x-15}{x^2+7x+10} : \dfrac{3x(x-2)+12}{x^3+8}$ $\qquad \left[\dfrac{x-3}{3}\right]$

94 $\left(\dfrac{x^2-3x-4}{x^2-8x+16} : \dfrac{2x^2+4x+2}{3x-12}\right) \cdot \left(\dfrac{x^2-x-2}{4x+x^2+3} : \dfrac{18x}{6x+18}\right)$ $\qquad \left[\dfrac{x-2}{2x^2+2x}\right]$

95 $\dfrac{a-2b}{a+2b} : \dfrac{a}{a^2-4b^2} \cdot \dfrac{a-2b}{a^3-6a^2 b+12ab^2-8b^3}$ $\qquad \left[\dfrac{1}{a}\right]$

96 $\dfrac{3x+9}{x^2+2x-3} : \dfrac{x^2-9x+8}{x^2+1-2x} : \left(\dfrac{3x+6}{x-8}\right)^2$ $\qquad \left[\dfrac{x-8}{3(x+2)^2}\right]$

97 $\dfrac{9y^2-81}{4y^3-196y} : \dfrac{3y+9}{2y^2-14y} : \left(-\dfrac{3y-9}{2y+14}\right)$ $\qquad [-1]$

98 $\dfrac{a^2-3a}{a^2+3a} : \left(\dfrac{2a-1}{12a^2-3} \cdot \dfrac{6a^2+3a}{a+3}\right)$ $\qquad \left[\dfrac{a-3}{a}\right]$

99 $\dfrac{7a+7x}{3a-3x} : \dfrac{14a+14x}{9x^2-9a^2} \left(\dfrac{3a^2-9a+3ax-9x}{a^2-4a+3}\right)^{-1}$ $\qquad \left[\dfrac{1-a}{2}\right]$

100 $\dfrac{y^3+9y-6y^2}{y^3-27} : \left(\dfrac{xy-3x+y-3}{xy-2x+y-2} : \dfrac{xy^2+3xy+9x+y^2+3y+9}{y^3-4y^2+4y}\right)$ $\qquad \left[\dfrac{x+1}{y-2}\right]$

101 $\dfrac{xy - 4y - 2x + 8}{xy - 3y - 2x + 6} \cdot \dfrac{4 - y^2}{x^2 - 7x + 12} : \dfrac{xy + 2x}{x^2 - 6x + 9}$ $\qquad \left[\dfrac{2 - y}{x}\right]$

102 $\left(\dfrac{9x^2 - 24x + 16 - 4y^2}{3x^2 - x - 4 - 2yx - 2y} : \dfrac{6x - 8 + 4y}{x^2 - 1}\right) : \left(\dfrac{1}{x} - 1\right)$ $\qquad \left[-\dfrac{1}{2}x\right]$

103 $\dfrac{5a^2 - 1 - 4a^4}{2a^2 + 3a - 2} : \left(\dfrac{2a^3 + a^2 - 2a - 1}{1 - a} \cdot \dfrac{a^2 - 1}{a + 2}\right)$ $\qquad \left[\dfrac{1}{a + 1}\right]$

104 $\left(\dfrac{3x^2 + 4x + 1}{2x^2 - 2} : \dfrac{3x^2 + 4x + 1}{x + 1}\right) \cdot \left(\dfrac{x^2 - 1}{x} : \dfrac{x^2 - 2x - 3}{x^2}\right)$ $\qquad \left[\dfrac{x}{2(x - 3)}\right]$

105 $\dfrac{x^3 - 3x - 2}{x^3 - x^2 - 2x} : \left(\dfrac{4x^2 + 3x - 1}{ax^2 + ax} \cdot \dfrac{a^2}{4x - 1}\right)$ $\qquad \left[\dfrac{x + 1}{a}\right]$

Semplifica le seguenti espressioni.

106 $\dfrac{1}{x - 3}\left(x + \dfrac{1}{x - 3}\right) - \dfrac{1}{x - 2}\left(x - \dfrac{1}{2 - x}\right) - \dfrac{x^2(x - 6) + 13x - 11}{(x^2 - 5x + 6)^2}$ $\qquad \left[\dfrac{1}{x^2 - 5x + 6}\right]$

107 $\left\{\left[-\left(\dfrac{x}{y^2 - x^2} + \dfrac{x}{x^2 + y^2}\right) : \dfrac{2x}{x^3 - x^2y + xy^2 - y^3} - \dfrac{x^2}{x + y}\right] - \dfrac{y^2}{x + y}\right\} : \dfrac{x^2}{x + y}$ $\qquad [-1]$

108 $\dfrac{2}{3}b + \dfrac{a^6 - b^6}{3a - 3b} : \dfrac{a^4 - b^4}{2a^2 + 2b^2} \cdot \dfrac{(b - a)^2}{(a^2 + b^2)^2 - a^2b^2}$ $\qquad \left[\dfrac{2}{3}a\right]$

109 $\left[\left(\dfrac{ab + 5}{ab + 1} - 1\right) : (ab) - \left(\dfrac{1}{ab - 1} - \dfrac{1}{ab}\right) - \dfrac{2}{1 - a^2b^2}\right]\left(\dfrac{10}{ab}\right)^{-1}$ $\qquad \left[\dfrac{1}{2(ab + 1)}\right]$

110 $\left(\dfrac{1}{ab} + \dfrac{1}{ab^2} - \dfrac{1}{a^2b}\right) \cdot \dfrac{a^2b^2 - a^2b + ab^2}{a^2b^2 - a^2 - b^2 + 2ab} - \dfrac{1}{b}$ $\qquad \left[\dfrac{1 - a}{ab}\right]$

111 $\left\{\left[\dfrac{x^4 - y^4}{x^2 + y^2} : \dfrac{(x - y)^2}{x^2 - y^2}\right] \cdot \dfrac{1}{x^2 + y^2 + 2xy}\right\}^3$ $\qquad [1]$

112 $\dfrac{x + 3}{7x - 7x^2 + 42} : \left(\dfrac{x - 3}{x + 3} : \dfrac{x^2 - 6x + 9}{x^2 + 6x + 9}\right)$ $\qquad \left[-\dfrac{1}{7(x + 2)}\right]$

113 $\left(-\dfrac{6}{y} + 1 + \dfrac{9}{y^2}\right)\left(\dfrac{1}{y^2 - 4y + 3} + \dfrac{1}{y - 3}\right)^2$ $\qquad \left[\dfrac{1}{(y - 1)^2}\right]$

114 $\left(\dfrac{a}{a - b} - \dfrac{b}{a + b} + \dfrac{a^2 + b^2}{a^2 - b^2}\right)\left(\dfrac{a^2 - b^2}{a^2 + b^2} - 1\right)$ $\qquad \left[-\dfrac{4b^2}{a^2 - b^2}\right]$

115 $x\left(1 + \dfrac{y + 2}{y - 2}\right) + a\left(1 - \dfrac{y + 2}{2 - y}\right)$ $\qquad \left[\dfrac{2y(a + x)}{y - 2}\right]$

116 $\left(\dfrac{a}{a - b} - \dfrac{b}{a + b} + \dfrac{a^2 + b^2}{a^2 - b^2}\right) : \left(a - b + \dfrac{a^2 + 3b^2}{a + b}\right)$ $\qquad \left[\dfrac{1}{a - b}\right]$

117 $\left(\dfrac{x + 2y}{x - 2y} + \dfrac{x - 2y}{x + 2y} - \dfrac{2x^2 + 1 + 4y^2}{x^2 - 4y^2}\right) : \dfrac{4y^2 + 1 + 4y}{3x^2 - 12y^2}$ $\qquad \left[\dfrac{3(2y - 1)}{2y + 1}\right]$

118 $\left[\dfrac{2(4x-3x^2-1)}{1-6x+9x^2}+\dfrac{3x-2}{3x-1}\right]:\dfrac{x^2}{9x^2-1}\cdot\left[x\cdot\left(\dfrac{1}{3x+1}-\dfrac{3x}{9x^2+6x+1}\right)\right]$ $\left[\dfrac{1}{3x+1}\right]$

119 $\left[\left(\dfrac{x-2}{2-5x}+\dfrac{3x+1}{x}+\dfrac{2x^2-x-2}{5x^2-2x}\right)\left(\dfrac{2-4x}{x}\right)^{-2}-\dfrac{x}{5x-2}\right]\cdot\dfrac{4x^2-1}{6x}$ $\left[\dfrac{2x+1}{3(5x-2)}\right]$

120 $\dfrac{x^3-3x^2}{x^2-4}\cdot\dfrac{x^2-x-2}{x^2-3x}\cdot\dfrac{x+2}{x^2+x}+\left[\dfrac{2x+y}{x^2-xy}\cdot\left(\dfrac{3x}{2x+y}-1\right)\right]:\dfrac{1}{x}$ $[2]$

121 $\dfrac{x^2-4}{x^2-1}:\left(\dfrac{2}{3x}-\dfrac{1}{x+1}\right)\left(\dfrac{1}{x}+\dfrac{1}{x+2}\right)+\left(\dfrac{y-2}{yx-y+x-1}-\dfrac{y+2}{yx-y-x+1}\right)\left(\dfrac{1}{y}-y\right)$ $\left[\dfrac{6x}{1-x}\right]$

122 $\left(\dfrac{x}{x-y}+\dfrac{6xy}{x^2-y^2}\right):\left(\dfrac{x}{2x+2y}-\dfrac{2x}{3x-3y}\right)-1$ $[-7]$

123 $\left[\left(\dfrac{3x-1}{y+2}\right)^3:\dfrac{(9x^2-6x+1)^2}{(y^2+3y+2)^3}+2y^2+2+4y\right]:\dfrac{(y+1)^2(x-1)}{6x-2}-\dfrac{2(6x-1)}{x-1}$ $\left[\dfrac{2y}{x-1}\right]$

124 $\left[\left(\dfrac{x^2-8x+18}{x^2-6x+8}-\dfrac{1}{x-4}\right):\dfrac{3ax-15a}{xy^2+y}-\dfrac{y\left(\dfrac{1}{2}x+2y\right)}{3ax-6a}\right]\cdot\left(\dfrac{2a}{2y-1}\right)^2-\dfrac{ay}{6y-3}$ $\left[\dfrac{ay}{3(2y-1)}\right]$

125 $\left[\left(\dfrac{x+2}{a-1}\right)^3:\dfrac{x^2+4+4x}{a^3-a^2-a+1}-2x-4\right]\left[\dfrac{x+1}{3x^2+7x+2}\cdot\left(a-\dfrac{a-2ax}{x+2}\right)\cdot\dfrac{x^2-4}{ax+a}\right]\cdot\dfrac{1}{2-x}$ $\left[\dfrac{a-3}{a-1}\right]$

126 $\left[\left(\dfrac{x+1}{x^2-16}\right)^{-1}-\dfrac{5x^2-4+x^3}{x^2+5x+4}\right]:\left(-\dfrac{2}{x+4}\right)\left(\dfrac{1}{x+6}-\dfrac{2}{x+10}\right)+\dfrac{x}{2x+2}$ $\left[-\dfrac{1}{x+1}\right]$

127 $\left[\left(\dfrac{a^2+b^2}{2ab}+1\right)^2:\dfrac{a+b}{a^2-3ab-4b^2}\right]\cdot\left(-\dfrac{2ab}{a+b}\right)^3-2ab(4b-a)(a+b)$ $[0]$

128 $\left[\dfrac{x-4y}{15x^2-43xy+30y^2}\cdot\left(\dfrac{8}{7}-\dfrac{y}{2x-y}+\dfrac{x}{x-4y}\right)-\dfrac{y}{4x^2-y^2}\right]\cdot(14x+7y)$ $\left[\dfrac{4x-5y}{2x-y}\right]$

129 $\left(\dfrac{3a}{a^2+y}-\dfrac{1}{a}+\dfrac{4a+2+y}{a^3+ay}\right):\dfrac{a^2+3a+2}{a^3}-2-\dfrac{4a}{a^3+ay+2a^2+2y}+\dfrac{2a}{a^2+y}$ $\left[-\dfrac{2y}{a^2+y}\right]$

130 $\left[\dfrac{2x^3}{(x+2)^3}+\dfrac{12ax^2+24ax+16a}{(ax+2a)(x^2+4x+4)}\right]\cdot\dfrac{x^2+x}{x^2-4xy-5y^2}\cdot\left(1+\dfrac{y}{x}\right)-\dfrac{2}{x-5y}$ $\left[\dfrac{2x}{x-5y}\right]$

131 $\left[\left(\dfrac{a}{a^2-b^2}-\dfrac{a}{a^2+b^2}\right):\dfrac{2a}{a^3-a^2b+ab^2-b^3}-\dfrac{a^2}{a+b}\right]-\dfrac{b^2}{a+b}$ $\left[-\dfrac{a^2}{a+b}\right]$