

# Esercizi di consolidamento

*Semplifica le seguenti espressioni togliendo le parentesi in una sola volta.*

**1**  $a^2 - 1 + \left\{ -x^2 - \left[ \frac{1}{2}a^2 + \left( -x^2 + \frac{1}{2}a^2 \right) + 1 \right] + 2a^2 \right\}$   $[2a^2 - 2]$

**2**  $\frac{1}{5}a^2b - \left[ 3x + a^2b - \left( \frac{1}{3}x + \frac{1}{2}a^2b \right) \right] + \frac{8}{3}x$   $\left[ -\frac{3}{10}a^2b \right]$

**3**  $\left( \frac{5}{6}ab - \frac{1}{4}a^2b + \frac{3}{5}a^2b^2 \right) - \left( \frac{1}{3}ab + \frac{3}{4}ab^2 - \frac{1}{4}a^2b \right) - \left( \frac{5}{2}ab^2 + \frac{1}{10}a^2b^2 \right)$   $\left[ \frac{1}{2}a^2b^2 - \frac{13}{4}ab^2 + \frac{1}{2}ab \right]$

**4**  $\left( \frac{1}{2}a^4 + 3a^3b^2 - \frac{5}{4} + 3a \right) - \left( -\frac{3}{5}a^4 - 1 + \frac{13}{6}a \right) + \left( -\frac{4}{3}a^3b^2 + \frac{1}{4} - \frac{5}{6}a \right)$   $\left[ \frac{11}{10}a^4 + \frac{5}{3}a^3b^2 \right]$

**5**  $\left( \frac{1}{4} + 2x^2 + a \right) - \left( \frac{3}{5}a^2x - \frac{1}{3}a \right) - \left[ -\left( -2x^2 + \frac{1}{2} \right) + \left( \frac{1}{3}a^2x - \frac{1}{6}a \right) \right]$   $\left[ \frac{3}{4} + \frac{3}{2}a - \frac{14}{15}a^2x \right]$

**6**  $(2x - 3y) - \left\{ -\left[ x^2 - (2x - y) + \left( 5y - \frac{3}{4}x^2 \right) \right] + \left( 2x + \frac{3}{2}y \right) \right\}$   $\left[ \frac{1}{4}x^2 - 2x + \frac{3}{2}y \right]$

**7**  $\left( a^2 + \frac{2}{3}b^2 + c^3 \right) - \left[ +\left( \frac{1}{3}a^2 - \frac{1}{3}b^2 + \frac{3}{4}c^3 \right) - \left( \frac{1}{6}a^2 + \frac{1}{2}b^2 - \frac{1}{4}c^3 \right) \right]$   $\left[ \frac{5}{6}a^2 + \frac{3}{2}b^2 \right]$

**8**  $\left( \frac{5}{12}x^2yz + \frac{19}{9}xy^2 + \frac{1}{10}x^3yz \right) - \left( -\frac{20}{9}xy^2z \right) - \left( \frac{4}{5}x^3yz + \frac{5}{3}xy^2z \right) - \left[ \left( \frac{1}{2}x^2yz + \frac{1}{9}xy^2 + \right. \right.$   
 $\left. \left. - \frac{3}{10}x^3yz \right) + \left( \frac{5}{9}xy^2z - \frac{3}{4}x^2yz - \frac{2}{5}x^3yz + \frac{2}{3}x^2yz \right) \right]$   $[2xy^2]$

**9**  $\left( \frac{4}{3}yz^2 - \frac{1}{3}yz + \frac{2}{3}x^2z^3 \right) - \left( xy - \frac{1}{2}y^2 - \frac{2}{5}yz + \frac{3}{20}y^2 \right) - \left[ \left( \frac{2}{3}yz^2 + \frac{4}{5}yz^2 \right) - \left( \frac{2}{15}yz - x^2z^3 + \right. \right.$   
 $\left. \left. + \frac{1}{3}x^2z^3 \right) - \left( -\frac{1}{5}yz \right) - \left( xy + \frac{1}{4}y^2 - \frac{3}{5}y^2 \right) \right]$   $\left[ -\frac{2}{15}yz^2 \right]$

**10**  $\frac{5}{2}x^2y - \frac{1}{4}xy + \frac{1}{2}xy^3z - \frac{3}{4}x^2y - \left( \frac{1}{4}xy + \frac{1}{12}xy^3z \right) + \left[ \left( \frac{1}{3}xy^3 - 2x^4y - \frac{3}{8}x^2y \right) - \left( \frac{3}{8}x^2y + x^2y \right) \right] +$   
 $- \left( \frac{5}{12}xy^3z - \frac{1}{2}xy + \frac{1}{3}xy^3 - 3x^4y \right)$   $[x^4y]$

**11**  $\left( yz^2 + \frac{1}{2}xy + x^2y^2 \right) - \left[ \left( \frac{1}{2}x^2y^2 + \frac{2}{3}x^2y \right) + \left( \frac{2}{15}x^2z + \frac{1}{5}x^2y^2 + \frac{2}{3}yz^2 \right) \right] - \left[ \left( \frac{1}{3}yz^2 - \frac{2}{9}x^2y + \right. \right.$   
 $\left. \left. + \frac{1}{4}xy \right) + \left( \frac{3}{10}x^2y^2 - \frac{1}{3}x^2y \right) - \left( \frac{2}{15}x^2z - \frac{1}{4}xy \right) \right]$   $\left[ -\frac{1}{9}x^2y \right]$

**Semplifica le seguenti espressioni contenenti prodotti notevoli.**

- 12**  $[(2y - 1)^2 + (0,5x + 2y - 1)(0,5x - 2y + 1)] : \left(\frac{1}{8}x^2\right)$  [2]
- 13**  $\left[(3a^2b - 4ab^2)^2 - \left(3a^2b + \frac{1}{2}ab^2\right)\left(3a^2b - \frac{1}{2}ab^2\right) + 24a^3b^3\right] : (-2ab)$   $\left[-\frac{65}{8}ab^3\right]$
- 14**  $\{[(-2x + 3y)(2x + 3y)(4x^2 + 9y^2) + 16(x^2 - 2y^2)(x^2 + 2y^2)] : (-17y^2)\}^2$   $[y^4]$
- 15**  $(2x - y)^2 + (2x + y)^2 - [(3y - 2x)(3y + 2x) - 9y^2] : (-2x) - x(8x - 2)$   $[2y^2]$
- 16**  $\left[\left(a - \frac{1}{3}b\right)^2 - a^2\right] : \left(-\frac{1}{3}b\right)\left(2a + \frac{1}{3}b\right) + \left(-\frac{2}{3}\right)\left(\frac{9}{4}a^2 - \frac{1}{6}b^2\right)$   $\left[\frac{5}{2}a^2\right]$
- 17**  $(2x - 1)(2x + 1) + (6x^2y - 3xy) : (-3xy) + (12x^3y^2 - 6x^2y^3 + 8x^4y^2) : (-2x^2y^2)$   $[3y - 8x]$
- 18**  $\left[\left(\frac{2}{3}x + \frac{9}{4}y\right)^2 - \left(-\frac{9}{4}y\right)^2\right] : \left(\frac{2}{3}x\right) - \left(\frac{1}{3}x + 1\right)^2 + 1$   $\left[\frac{9}{2}y - \frac{1}{9}x^2\right]$
- 19**  $\left[\left(10x^2y^3 - \frac{1}{3}x^4y^2 + 5x^5y^3\right) : \left(\frac{5}{3}x^2y^2\right) - 3y(2 - x^3)\right] : \left(-\frac{1}{5}x^2\right) + 30xy$  [1]
- 20**  $[(3x + 2y)(2x - 3y) - 6(x - y)(x + y)] : (-5xy) + 8 - \left(\frac{2}{3}x - 3\right)\left(\frac{2}{3}x + 3\right)$   $\left[18 - \frac{4}{9}x^2\right]$
- 21**  $\left\{-\left[\frac{1}{5}x(5 + 10x) - 7x^6 : \left(\frac{7}{2}x^4\right)\right] \cdot 2x^3 - 4 + (x^2 + 2)^2\right\} : (-2x)^2$   $\left[1 - \frac{1}{4}x^2\right]$
- 22**  $[(2x + 3)(4x^2 - 6x + 9) - (2x + 3)^3] : (-18x) + (x - 1)(x + 2) - 1$   $[x^2 + 3x]$
- 23**  $\{(-2a)^5 : (2a)^4 + (a + 2)^2 - 2[(-2a)^3]^2 : [(-2a)^2]^2 - 2(a + 2)\}^2 - (-7a)^2$   $[49a^4 - 49a^2]$
- 24**  $\left(\frac{3}{4}a^2x - \frac{1}{2}ax\right) : \left(-\frac{1}{2}ax\right) + \left(\frac{1}{5}a^3x^2 - \frac{1}{2}a^2x^3 - \frac{2}{5}a^2x^2\right) : \left(+\frac{2}{5}a^2x^2\right)$   $\left[-a - \frac{5}{4}x\right]$
- 25**  $\left\{\left[\left(2a^2b^3 - \frac{1}{4}a^5\right)(-4a) + (2ab + 1)(4a^2b^2 - 2ab + 1)\right]^2 - 1\right\} : a^6$   $[a^6 + 2]$
- 26**  $[y^3 + 3y(x - 1)(x - 1 + y)] : (-2y) + 3\left[\left(x + \frac{1}{2}y\right)\left(\frac{1}{2}x - 1\right) + \frac{1}{2}\right]$   $\left[-\frac{1}{2}y^2 - \frac{3}{4}xy\right]$
- 27**  $\left\{\left[(x^2 + y^2)^2 + x^2y^2 - 2xy(x^2 + y^2) - (x^4 + y^4)\right] : (-xy) - 2(x^2 + y^2)\right\} : (-xy)$  [3]
- 28**  $\{[(2a + t)^3 - (2a + t)(4a^2 - 2at + t^2)] : (-6at) + t\} : (-2a)$  [1]
- 29**  $\{[(x - y)^2(x + y) - (x - y)(x + y)^2] : (-2y) + y^2\}^2 : (-x)^4$  [1]
- 30**  $[(a + 2b)^2 + 4(a^2 - 3b^2) - 2(a + 2b)(a - 2b) + 3a(-a + 3b)] : \left(-\frac{13}{5}ab\right)$   $[-5]$
- 31**  $\left[3x^2 - \frac{2}{3}x\left(3x - \frac{3}{4}y\right) - \frac{1}{3}y(x + 6y)\right](x - 3y) - (x - 6y)(x^2 - y^2)$   $\left[\frac{19}{6}x^2y - \frac{3}{2}xy^2\right]$
- 32**  $[(2a - b)(2a + b) + 1]^2 - (a^2 + b^2 - 1)^2 - 5a^2(3a^2 + 2)$   $[-10a^2b^2]$

$$33 \quad (2a + 3b - 1)(2a - 3b + 1) - 2[(a - 2)(a + 2) + (b - 2)^2] + (3b + 2)^2 - 2(a - b)(a + b) \quad [26b + 3]$$

$$34 \quad (a - 2x + 1)(a + 2x - 1) + (2x - 1)^2 + a^2(2a + 1)(2a - 1) \quad [4a^4]$$

$$35 \quad [(x - 2)(x + 2)(x^2 + 4)]^2 - [(2x - 1)(2x + 1)(4x^2 + 1)]^2 - 20(3 - 4x^4)(3 + 4x^4) \quad [65x^8 + 75]$$

**Semplifica le seguenti espressioni riassuntive.**

$$36 \quad \left(\frac{3}{2}a + b\right)\left(\frac{1}{2}b + 1\right) + a\left(\frac{5}{2} + a\right) - \left[(a - b)(a + 2b - 1) + \left(-\frac{1}{4}ab\right)\right] \quad \left[5a + \frac{5}{2}b^2\right]$$

$$37 \quad \left(y - \frac{2}{3}\right)\left(\frac{1}{2}x + 1\right) + \frac{2}{3} - \left[(2y + 1)\left(\frac{5}{2} + \frac{1}{4}x\right) + \left(1 - \frac{5}{12}\right)(-x)\right] \quad \left[-4y - \frac{5}{2}\right]$$

$$38 \quad \frac{1}{3}a(3a - 5) + a^2b^2 + (ab - 1)\left(\frac{2}{3}ab + \frac{3}{4}\right) - \left[\left(a + \frac{1}{3}\right)(a - 2) + \frac{1}{12}(5ab + 4)(4ab - 3)\right] \quad \left[\frac{11}{12}\right]$$

$$39 \quad (a + b)\left(\frac{1}{2}a - \frac{1}{2}b + 2\right) - (2b + 1)\left(\frac{1}{4}a + 3\right) + \left(-\frac{1}{2}\right)(a + b)(a - b) + 4b + 3 \quad \left[\frac{7}{4}a - \frac{1}{2}ab\right]$$

$$40 \quad \frac{4}{3}a^2 + (x - 3a)(x + 2a) - \left(\frac{1}{2}x^2 - a\right)\left(\frac{2}{3}a + 1\right) - \left[(x + a)\left(\frac{1}{2}x - 3a\right) - \frac{1}{3}a(x^2 - 3)\right] \quad \left[\frac{3}{2}xa - a^2\right]$$

$$41 \quad \left(x - \frac{1}{2}a\right)(x + 1) - ax - \left[\frac{1}{2}(2x + a)(x - a) + (x + 1)(-ax)\right] - a\left(x^2 - \frac{1}{2}\right) \quad \left[x + \frac{1}{2}a^2\right]$$

$$42 \quad (x + y)\left(\frac{2}{3}z - 3\right) + x\left(-\frac{8}{3}z + 1 + 4x\right) - \left[2(2x + z)(x - z) - 2x + \frac{2}{3}y(z - 3)\right] \quad [2z^2 - y]$$

$$43 \quad (a + x)(a - x) + \left(2a - \frac{1}{2}x\right)^2 - \left[\frac{1}{4}(5a - x)(4a + x) - 9a - \frac{1}{2}(x - 1)^2\right] - 9a\left(1 - \frac{1}{4}x\right) \quad \left[\frac{1}{2} - x\right]$$

$$44 \quad \left\{(3x + a)(x - a) + a^2(1 - 2x^2) - [(x + 1)(3x - 1) - (-2xa)^2] + 2x(a + 1)\right\}(2a^2x^2 - 1) \quad [4a^4x^4 - 1]$$

$$45 \quad \left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right) - 5ax + (a + 1)^2 - \left[(2x - a)^2 - (x - 1)\left(\frac{2}{3}x + 2a\right) - \frac{1}{3}x(7x + 2)\right] \quad \left[ax + \frac{3}{4}\right]$$

$$46 \quad \frac{1}{4}x^2y + (6y + 2)\left(1 + \frac{4}{3}y^2\right) - \left\{\left(\frac{1}{2}xy + 1\right)\left(2 + \frac{1}{2}x\right) - \left[(-2y)^3 + \frac{1}{2}x + (-3y)\left(\frac{8}{9}y + 2\right)\right]\right\} \quad [-xy]$$

$$47 \quad \left(ax - \frac{1}{2}b + 1\right)\left(ax - 1 + \frac{1}{2}b\right) - \left(ax - 1 + \frac{1}{2}b\right)^2 - (2 - b)\left(ax - 1 + \frac{1}{2}b\right) \quad [0]$$

$$48 \quad \left(1 + \frac{5}{4}ay^2\right)(y - 2) + y\left[x\left(\frac{1}{8}y^2 + a\right) + y\left(\frac{5}{2}a - 1\right)\right] - \frac{11}{8} - \left[(x + 2a)\left(\frac{1}{2}y\right)^3 + (ay - 1)(y^2 + x) + \left(-\frac{3}{2}\right)^3\right] \quad [x + y]$$

$$49 \quad 4\left(\frac{2}{3}y^2 - \frac{1}{2}\right) + (4xy - 1)^2 + x[1 - x(4y^2 + 9) - y] - \left[\left(\frac{9}{2}x^2 + 1\right)\left(\frac{8}{3}y^2 - 2\right) - xy\right] \quad [x - 8xy + 1]$$

$$50 \quad -\frac{1}{2}ax + \left(\frac{1}{3}x - \frac{1}{2}a\right)\left(\frac{3}{2}a - \frac{4}{3}bx\right) - \left[\left(\frac{1}{4}a + 3x\right)\left(-3a + \frac{8}{3}bx\right) - b\left(\frac{2}{3}x\right)^2\right] - [3x(3a + 2bx) + \\ -2b(x - 2)(x + 2)] \quad [-8b - 12bx^2]$$

$$51 \quad [(a + b)(a + 2b - 1) - 3ab]\left[\frac{1}{2}a - (a + 1)(b - 2)\right] - \frac{1}{2}a^3(5 - 2b) + \frac{1}{2}(a + 5b)(a - 2b) + \\ + 2b^2(b - 1)(a + 1) + 2(b + 1)^2 + 2a(1 - 2b^2) \quad [2b + 2]$$

$$52 \quad (x + 2a)^3 + \left(\frac{3}{4}x^2 - 1\right)\left(2a - \frac{1}{2}x + 2\right) - \left[\left(\frac{1}{2}x + a\right)^2\left(\frac{5}{2}x + 8a + 6\right) - 6\left(a + \frac{1}{2}\right)^2 + 3ax\left(x + \frac{1}{2}a - 2\right)\right] \\ \left[4a + \frac{1}{2}x - \frac{1}{2}\right]$$

$$53 \quad 3(x^2 + 1) + (x - 2)(x - 1) + x(ax - 1)(ax - 3) - [2(x - 2)^2 + x(ax - 3)^2 + (2x + 5)(x - 1)] \\ [2ax^2 - 4x + 2]$$

$$54 \quad [(x - a)(x + a) - (x - a)^2 + (2a)^2]^2 - \{(2a^2 + x^2)^2 - x[(-2a)^3 + x^3 + 2]\} \quad [2x]$$

$$55 \quad (ax - 1)^3 - (x + 2)^3 - \left[(ax + 3)(2ax - 3) - x(x + 4)(x + 3) + (2ax)^2\left(\frac{1}{4}ax - \frac{3}{2}\right)\right] - x^2(a - 1)(1 + a) \\ [2x^2]$$

$$56 \quad \left(\frac{1}{3}ax - \frac{2}{3}\right)^2(-9a) + (x + a^2)(x - a^2) - [(a^2 - 1)(4x - a^2) - 4a + x^2(1 - a^3)] \quad [4x - a^2]$$

$$57 \quad 2[(a^3 - x)^2 + 2a^2(2 + ax) - (x - 2a)(2a + x)] - [(a + x - 1)(a + 1 - x) + (x - 1)^2]^3 - (-4a)^2 \quad [a^6]$$

$$58 \quad \left(x + \frac{1}{2}\right)(x^2 + 2x + 2) - \left[\left(x - \frac{1}{3}\right)\left(x + \frac{1}{3}\right) - \frac{1}{3}(5ax - 2a^2) + \frac{3}{4}\right] - \left[\left(x + \frac{1}{2}\right)^3 + x + \right. \\ \left. - \frac{1}{3}(2a + x)(a - 3x) + \frac{1}{9} - \left(x - \frac{1}{2}\right)^2\right] \quad \left[\frac{1}{4}x + \frac{3}{8}\right]$$

$$59 \quad (x + y + z)^2 - (x - y - z)^2 - 2x^2(y + z) + (x - y)^3 - \{4xz - 2x(-2y + xz + 2y^2) + \\ - [(x^2 - 3y)(x^2 - 2y) + y(5x - y)(2x - y)]\} \quad [x^3 + x^4 + 6y^2]$$

$$60 \quad \left[\left(x + \frac{1}{3}a\right)^2 - \frac{1}{9}a^2\right]\left(x^2 - \frac{2}{3}ax\right) + [(ax + 1)(ax - 2) - a^2x^2]^2 - [(x - a)(a + x)(2a^2 + x^2) + \\ + 4(ax + 1) - \left(-\frac{2}{3}ax\right)^2] \quad [2a^4]$$

$$61 \quad \left(x - \frac{3}{2}z\right)\left(x + \frac{1}{2}z\right) - \frac{2}{9}y(-2y + 6x - 3z) - 2z^2\left(z - \frac{1}{2}\right) - \left[\left(x - \frac{2}{3}y - \frac{1}{2}z\right)^2 + (x + z)^3 + \right. \\ \left. - (x^2 + z^2)(3z + 3x)\right] \quad [2x^3]$$

$$\begin{aligned} \text{62} \quad & (x-z)^2 + \left(\frac{1}{2}y + x - z\right)\left(\frac{1}{2}y + z - x\right) - \left[\left(x + \frac{3}{2}z\right)\left(x + \frac{1}{2}z\right) - (x+3z)(x-z) + \right. \\ & \left. + \left(\frac{1}{2}y - 2z\right)\left(\frac{1}{2}y + 2z\right)\right] \end{aligned} \quad \left[\frac{1}{4}z^2\right]$$

$$\text{63} \quad \left\{ \left[ \left(x + \frac{4}{5}y\right)\left(x - \frac{5}{4}y\right) + y^2 \right] : x \right\}^2 - \left(x - \frac{5}{4}y\right)^2 - \frac{2}{5}y\left(4x - \frac{7}{5}y\right) \quad \left[-\frac{4}{5}y^2\right]$$

$$\text{64} \quad (x+2a)^3 + (x+2a)(2x+a) - \left[\left(\frac{3}{2}x+a\right)^2 - \left(\frac{1}{2}x-a\right)\left(\frac{1}{2}x+a\right)\right] - x[(x+6a)(x+2a) - 2a(x-1)] \quad [8a^3]$$

$$\text{65} \quad \left[\left(\frac{3}{5}ab^2 - \frac{1}{10}ab\right) : \left(\frac{1}{2}ab - \frac{1}{5}ab\right)\right]^2 - (6ab-a)\left(2ab + \frac{1}{3}a\right) : (3a^2) \quad \left[\frac{2}{9} - \frac{4}{3}b\right]$$

$$\text{66} \quad \left\{ \left[ \left(\frac{1}{3}a-b\right)^2 \left(\frac{1}{3}a+b\right)^2 - b^4 \right] : \left(\frac{1}{9}a^2\right) + \left(2b - \frac{1}{3}a\right)\left(2b + \frac{1}{3}a\right) \right\} : \left(-\frac{1}{2}b\right) \quad [-4b]$$

$$\text{67} \quad [(a^2x^2+1)(a^2x^2-1)+1] : \left(\frac{1}{3}a^3x^3\right) + [(ax-1)(ax+a) - a(ax-x-1)] : (ax) \quad [4ax]$$

$$\text{68} \quad \{5[(ab)^3 : (a^2b^3)] + 2b\}^2 + \left[\left(a - \frac{2}{5}ab^2\right)(5a - 2ab^2) - 5a^2\right] : \left(\frac{1}{2}ab\right) - (a+2b)(5a+2b) \quad \left[20a^2 + \frac{8}{5}ab^3\right]$$

$$\text{69} \quad \left\{ [(x+2)^3 - 8] : x - \frac{1}{2}\left(\frac{1}{2}x+3\right)(x+4) - 6 \right\} : \left(\frac{1}{4}x\right) - (2x-3)^2 \quad [15x - 4x^2 + 5]$$

$$\text{70} \quad [(ab-ac)^2(bc)^2] : \left[(abc-1)^2 + \frac{1}{2}(2abc-1)(2abc+3) + \frac{1}{2}\right] + \frac{2}{3}bc \quad \left[\frac{1}{3}b^2 + \frac{1}{3}c^2\right]$$

$$\text{71} \quad \left[4a(1-ax) + \left(\frac{1}{3}ax - \frac{2}{3}\right)^2(-9a) + (x+a^2)(x-a^2) - x^2\right] : \left(-\frac{1}{2}a^3\right) - 2\left(x - \frac{1}{2}a\right)\left(x + \frac{1}{2}a\right) \quad \left[2a + \frac{1}{2}a^2\right]$$

$$\text{72} \quad \left[(ax+bx)\left(ax - \frac{1}{2}bx\right)^2 - \frac{1}{4}b^3x^3\right] : \left[\frac{1}{2}(xa)^5 : (2a^4x^2)\right] + (2b-3a)(3a+2b) \quad [b^2 - 5a^2]$$